COMPARISON OF TWO ELECTROMAGNETIC MODELS FOR MICROWAVE SURFACE CRACK/SLOT DETECTION USING OPEN-ENDED WAVEGUIDES

C. Huber, M. L. Peterson,* and J. Bourland†
Applied Microwave Nondestructive Testing Laboratory
Electrical Engineering Department
Colorado State University
Ft. Collins, CO 80523

* Advanced Inspection and Monitoring Laboratory
Mechanical Engineering Department
Colorado State University
† Department of Mathematics, Colorado State University

INTRODUCTION

Microwaves have been shown to be able to detect surface breaking hairline cracks on metal specimens [1]. A microwave signal is typically fed through a rectangular waveguide probe. The incident and reflected signals in the waveguide form a standing wave, whose characteristics change depending on the relative position of a crack and the rectangular waveguide probe. Two separate electromagnetic models have been developed to mathematically predict the crack characteristic signal, i.e. the variation of the measured standing wave in the waveguide when it is scanned across a surface breaking crack. These models can be used to optimize measurement parameters. One model uses a mode matching approach, whereas the other model involves a moment solution approach. This paper presents a comparision of these two methodologies, which demonstrates the major advantages in the use of a moment solution approach. The most important result shown is that a moment solution approach is more general, eliminating the destinction between a crack being at the edge or in the middle of a rectangular waveguide probe. The convergence behavior is also studied for both methodologies. Faster convergence is observed using the moment solution approach. Finally, a moment solution approach allows for an easy expansion of the electromagnetic model to the analysis of finite cracks, and can be more readily expanded to encompass covered cracks as well.

TECHNICAL APPROACH

The objective of the modeling effort described is to evaluate the electric and magnetic field distribution at an arbitrary location inside the system formed by a waveguide and a crack. This system is modeled as a junction of two waveguides. In order to evaluate the fields the reflection coefficients of the dominant (incident) mode and the higher-order modes must be evaluated. In order to study the two different mathematical approaches, i.e. a mode matching approach and a moment solution approach, an arbitrary relative position of the crack with respect to the waveguide is considered. Figures 1a-b show the relative geometry of a crack with a width of w, a depth of d and a length of ℓ, and an open-ended waveguide with dimensions a and b with the long crack axis parallel to the broad dimension of the waveguide. The distance δ is a dimension indicating the location of the crack relative to a reference location in the narrow dimension of the waveguide aperture, b. It is assumed that the crack may be filled with a dielectric, which allows the consideration of cracks filled with rust, paint, dirt, ice, etc. The crack length is not necessarily equal to the broad waveguide dimension, thus allowing the analysis of finite cracks as well.

DERIVATIONS

The fields in the waveguide and the crack are described by their orthonormal mode vectors which form complete sets for describing their respective electromagnetic fields [2]. The i^{th} orthonormal mode vectors for the waveguide side (index a) are given by $\boldsymbol{e}_{ai}$ and $\boldsymbol{h}_{ai}$. Similarly, the i^{th} orthonormal modes in the crack side (index b) are given by $\boldsymbol{e}_{bi}$ and $\boldsymbol{h}_{bi}$. The orthonormal mode vectors satisfy an orthogonality relationship in the waveguide and the crack. For a general case it is assumed that all higher-order TE_{nm} and TM_{nm} modes are generated at the junction (i.e. z = 0, see Figure 1). Therefore, the total transverse electric and magnetic fields, in terms of the orthonormal mode vectors of the waveguide, are written as:

$$\boldsymbol{E}_{at} = \sum_i C_i e^{-\gamma_{ai} z} \boldsymbol{e}_{ai} + \sum_i A_i e^{\gamma_{ai} z} \boldsymbol{e}_{ai} \tag{1a}$$

$$\boldsymbol{H}_{at} = \sum_i C_i Y_{ai} e^{-\gamma_{ai} z} \boldsymbol{u}_z \times \boldsymbol{e}_{ai} - \sum_i A_i Y_{ai} e^{\gamma_{ai} z} \boldsymbol{u}_z \times \boldsymbol{e}_{ai} \tag{1b}$$

where $\boldsymbol{u}_z$ denotes the unit vector in the direction of propagation, and $\{C_i\}$ and $\{A_i\}$ are complex coefficients of the incident and reflected modes, respectively. The mode propagation constant in the waveguide is $\{\gamma_{ai}\}$, and $\{Y_{ai}\}$ is the modal characteristic admittance in the waveguide.

In the crack side the transverse fields can be similarly expanded in terms of the orthonormal modes as:

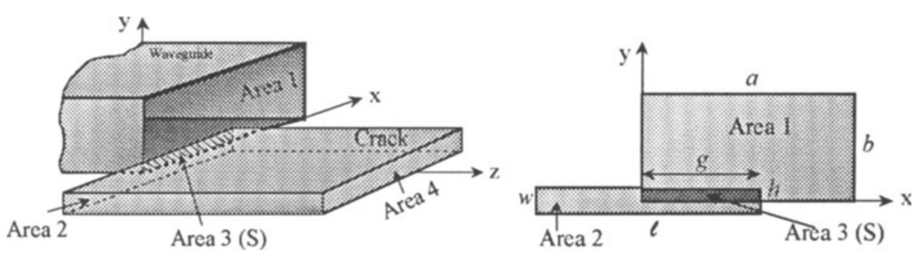

Figure 1: Relative geometry of a surface crack and a waveguide aperture; a) side view, b) plan view.

$$E_{bt} = \sum_{i} F_i e^{-\gamma_{bi}z} e_{bi} - \sum_{i} B_i e^{\gamma_{bi}z} e_{bi} \tag{2a}$$

$$H_{bt} = \sum_{i} F_i Y_{bi} e^{-\gamma_{bi}z} u_z \times e_{bi} + \sum_{i} B_i Y_{bi} e^{\gamma_{bi}z} u_z \times e_{bi} \tag{2b}$$

where $\{F_i\}$ and $\{B_i\}$ are the complex coefficients of the modes in the crack which propagate in the positive and negative z-direction, respectively.

The fields in the waveguide and the crack are defined by the solution of Maxwell's equations that satisfy all the boundary conditions except at the junction. Forcing the boundary conditions for the transverse fields at the aperture effectively allows for the solution of all of the unknown field coefficients. For a general case, as shown in Figure 1, the following boundary conditions must be satisfied. The tangential electric field components (denoted by subscript t) must vanish over the conducting surfaces. Both the tangential components of the electric and magnetic fields must also be continuous over the aperture S (Area 3) at $z = 0$.

Mode Matching Approach

To employ the mode matching approach, appropriate boundary conditions at the waveguide aperture are applied which depend on the relative position of the crack within the waveguide aperture [1,3]. The boundary value problem is solved by an expansion of the aperture fields over the modes in the waveguide and the crack. The field equations are satisfied through an expansion in terms of unknown coefficients and the eigenfunctions of the waveguide and the crack. These expansions are set up for each of the two domains (the waveguide and the crack) and the boundary conditions matched to satisfy continuity (Fourier Boundary Matching Approach) The resulting calculation time depends on the number of higher order modes considered. For a general crack of arbitrary dimensions (length ℓ, width w, and depth d), all higher order modes are generated at the junction z = 0. The smallest number of higher order modes necessary to produce a good theoretical crack characteristic signal depends on the crack dimensions, the waveguide dimensions, the operating frequency and the required accuracy. It is necessary to make a distinction between empty, filled and finite cracks. The nature of the solution is also dependent on the relative position of the crack within the probing waveguide aperture (i.e. in the middle or at the edge).

Forcing the boundary conditions for the crack at the edge of the waveguide aperture renders the following equations which may be solved simultaneously. From this solution the unknown coefficients and hence the field distribution at any point inside the waveguide may be determined. Therefore, one set of equations is formulated when the tangential electric field is forced to be zero over the Area 1 and continuous over the Area 3:

$$\sum_{i=1}^{L_a} (C_i + A_i) e_{ai} = \sum_{i=1}^{L_b} (F_i - B_i)\, e_{bi} \tag{3}$$

Another set of equations is obtained in a similar fashion by satisfying the tangential electric field to be zero over the Area 2 (i.e. the waveguide flange) and to again satisfy continuity over the aperture (Area 3):

$$\sum_{i=1}^{L_b}(F_i - B_i)e_{ai} = \sum_{i=1}^{L_a}(C_i + A_i)\, e_{bi} \tag{4}$$

A third set of equations satisfies the condition of continuity of the tangential magnetic field components over the aperture (Area 3):

$$\sum_{i=1}^{L_a}(C_i - A_i)\cdot \bar{u}_z \times e_{ai} = \sum_{i=1}^{L_b}(F_i + B_i)\cdot \bar{u}_z \times e_{bi} \tag{5}$$

Finally, to guarantee a vanishing electric field at the end of the crack (Area 4) a last set of equations results:

$$\sum_{i=1}^{L_b} F_i e^{-\gamma_{bi} d} e_{bi} - B_i e^{\gamma_{bi} d} e_{bi} = 0 \tag{6}$$

With this description it is possible to make use of mode orthogonality to derive a set of equations involving only the unknown coefficients.

Moment Solution Approach

The reflection coefficient at the waveguide aperture can be expressed with a generalized scattering matrix, which can be derived by using a moment solution approach [4]. In order to obtain a general representation of a system formed by a waveguide aperture and a metallic surface with a crack, arbitrary incident electric and magnetic fields in the waveguide are assumed with a moment solution approach. The incident and reflected fields in the waveguide and the crack are expressed in terms of their discrete orthonormal eigenfunctions (for the dominant mode and the higher-order modes) with unknown complex coefficients. These coefficients represent the amplitude and the phase of the respective eigenfunctions. A magnetic current density $\boldsymbol{M}$ is introduced over the common aperture (of the system) formed by the waveguide and the crack. The waveguide and the crack can then be separated into two parts, applying the equivalence principle [5]. The method of moments provides for a numerical solution for the complex field coefficients [6]. The degree of accuracy in approximating the electric and magnetic field distributions anywhere in the waveguide and in the crack subsequently depends upon the number of higher-order modes used, and on the appropriate choice of basis functions for the expansion of the magnetic current density. The convergence behavior is used to analyze all of these criteria. Finally, a generalized scattering matrix is formulated by forming the system of equations in a matrix form and solving for the reflection coefficient at the aperture of the waveguide [7]. This approach eliminates the restrictions of the previously developed model. This approach is more versatile since it is suitable for any incident mode, or any combination of different incident modes, as the excitation field. No distinction is necessary between exposed, filled, long and finite cracks. Additionally, the solution is independent of the crack location (i.e. in the middle or at the edge) within the waveguide aperture.

The total transverse electric and magnetic fields in the waveguide is now given by:

$$\boldsymbol{E}_{at} = \sum_i C_i e^{-\gamma_{ai} z} e_{ai} - \sum_i C_i e^{\gamma_{ai} z} e_{ai} + \sum_i D_i e^{\gamma_{ai} z} e_{ai} \tag{7a}$$

$$H_{at} = \sum_i C_i Y_{ai} e^{-\gamma_{ai} z} u_z \times e_{ai} + \sum_i C_i Y_{ai} e^{\gamma_{ai} z} u_z \times e_{ai} - \sum_i D_i Y_{ai} e^{\gamma_{ai} z} u_z \times e_{ai} \tag{7b}$$

where $\{C_i\}$ and $\{D_i\}$ are the respective coefficients of the incident modes and the modes produced by M. In the crack the total transverse fields are then given by:

$$E_{bt} = \sum_i B_i e^{-\gamma_{bi} z} e_{bi} - \sum_i B_i e^{\gamma_{bi} z} e_{bi} + \sum_i G_i e^{-\gamma_{bi} z} e_{bi} \tag{8a}$$

$$H_{bt} = \sum_i B_i Y_{bi} e^{-\gamma_{bi} z} u_z \times e_{bi} + \sum_i B_i Y_{bi} e^{\gamma_{bi} z} u_z \times e_{bi} + \sum_i G_i Y_{bi} e^{-\gamma_{bi} z} u_z \times e_{bi} \tag{8b}$$

with $\{B_i\}$ and $\{G_i\}$ as the respective coefficients of the reflected modes and the modes produced by *-M*. The last term in each of the above transverse field Equations (7) and (8) corresponds to the fields generated by the equivalent magnetic current density M. At $z=0$ these first two terms will cancel with each other if combined.

The moment solution approach and the subsequent generalized scattering parameters depend on the choice of an initially unknown equivalent magnetic current density M over a conducting surface. M is described by basis functions that form a complete set. Since M describes the physical behavior of the magnetic field over the aperture S, it is best to choose the basis functions to have similar properties to the orthonormal mode vectors of the transverse magnetic fields in the waveguide and the crack. In this way, a relatively fast convergence may be obtained. The number of basis functions used to describe M is critical for fast convergence. A thorough study of the convergence behavior for the crack at different relative positions with respect to the waveguide aperture is given by Huber [8].

After making appropriate substitutions for $\{D_i\}$ and $\{B_i\}$, these terms may be evaluated from boundary conditions which includes the applied magnetic current density M. The following equation is obtained:

$$2\sum_{i=1}^{L_a} C_i Y_{ai} P_{aik} = \sum_{i=1}^{L_a}\left(\sum_{j=1}^{N} V_j H_{aij}\right) Y_{ai} P_{aik} + \sum_{i=1}^{L_b}\left(\sum_{j=1}^{N} V_j H_{bij}\right)\frac{e^{2\gamma_{bi} d}+1}{e^{2\gamma_{bi} d}-1} Y_{bi} P_{bik} \tag{9}$$

The set of Equations (9) may also be written in a matrix form as:

$$\vec{I} = \left[\overline{Y}_a + \overline{Y}_b\right]\vec{V} \tag{10}$$

where:

$$\vec{I} = 2P_a^T Y_a \vec{C} \tag{11a}$$

$$\overline{Y}_a = P_a^T Y_a H_a \tag{11b}$$

$$\overline{Y}_b = P_b^T Y_b E_1 E_2 H_b \tag{11c}$$

Y_a, Y_b, E_1 and E_2 are diagonal matrices whose elements are readily evaluated. Note that $P_a^T = H_a^T$ and $P_b^T = H_b^T$ when using the Galerkin's method. Now, it is possible to evaluate the generalized scattering matrix S of the junction. Subsequently, S_{11} is expressed as:

$$S_{11} = \vec{A}\vec{C}^{-1} = 2H_a\left[\overline{Y}_a + \overline{Y}_b\right]^{-1} P_a^T Y_a - U \tag{12}$$

where U is the unity matrix. Note that S_{ij} is the amplitude of the i^{th} mode due to the j^{th} incident mode with unit amplitude.

CONVERGENCE

In order to show the speed of convergence for the mode matching approach, the normalized reflection coefficient of the higher-order modes is plotted in Figure 2. It can be seen, that for a crack located at the relative coordinates (0,-0.0008) more than 150 modes should be considered, as significant energy remains in those modes.

For a moment solution approach only 50 modes will have to be considered for a crack at the relative coordinates (0,-0.0008) (Figure 3). The energy in higher-order modes is significantly less than that observed in the mode matching method. The computational time is also much reduced when compared to the previous case.

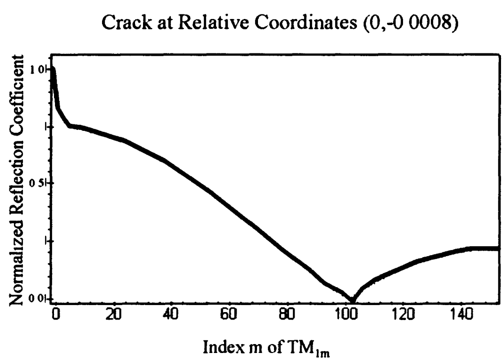

Figure 2: Reflection Coefficient of higher-order modes when employing a mode matching approach.

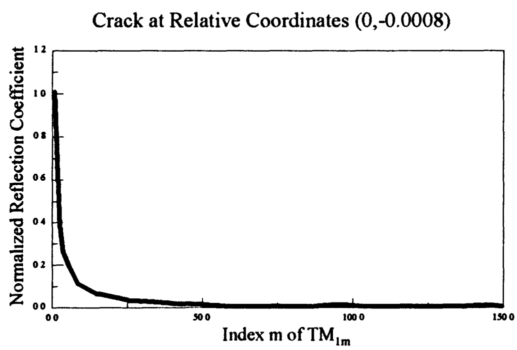

Figure 3: Reflection Coefficient of higher-order modes when employing a moment solution approach.

CONCLUSIONS

A theoretical model describing the electromagnetic properties in the interaction of fields propagating in a system formed by a waveguide and a crack in a metal surface was developed employing a mode matching approach [3]. However, this model possesses certain disadvantages. In particular, it is necessary to differentiate between a crack partially or fully within the waveguide aperture. This model also required a separate code for a finite crack, i.e. the crack is shorter than the broad dimension of the waveguide. Expanding this model to encompass the numerical evaluation of the electromagnetic fields for an arbitrary crack evaluated at arbitrary positions relative to the waveguide aperture is quite complex. Moreover, this model is not well suited for expansion to the evaluation of covered cracks. As an alternative, a moment solution approach has been employed to express the reflection coefficient for this system in terms of the generalized scattering matrix for an arbitrary incident field [8]. The evaluation of the generalized scattering matrix employs the equivalence principle and a moment solution approach. This approach has proven to be a powerful technique for theoretically evaluating the electromagnetic properties. An attractive feature of this model is its generality. With relatively slight modifications it can be applied to empty, filled and finite cracks. The solution is independent of the position relative to the waveguide aperture using the moment solution method. Thus, compared to the previously developed mode matching approach, this method is more flexible while simultaneously reducing the computational time. The moment method also shows promise as an appropriate approach for expansion to the evaluation of covered cracks.

REFERENCES

1. C. Yeh and R. Zoughi, "A Novel Microwave Method for Detection of Long Surface Cracks in Metals," *IEEE Trans. on Instrumentation and Measurement*, vol. IM-43, no. 5, pp. 719-725, Oct. 1994.
2. F. E. Borgnis and C. H. Papas, "Encyclopedia of Physics: Electromagnetic Waveguides and Resonators," *Springer Verlag*, vol. 16, 1958.
3. C. Yeh, "Detection and Sizing of Surface Cracks in Metals Using Open-ended Rectangular Waveguides," Ph.D. Dissertation, Electrical Engineering Department, Colorado State University, May 1994.
4. H. Auda and R. F. Harrington, "A Moment Solution for Waveguide Junction Problems," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-31, no. 7, pp. 515-519, July 1983.
5. R. F. Harrington, "Time Harmonic Electromagnetic Fields," *McGraw-Hill*, New York, 1961.
6. R. F. Harrington, "Field Computation by Moment Methods," *Krieger Publishing Co.*, 1982.
7. R. F. Harrington and J. R. Mautz, "A generalized network formulation for aperture problems," *IEEE Trans. Antennas Propagation*, vol. AP-24, no. 6, pp. 870-873, Nov. 1976.
8. C. Huber, "Electromagnetic Modeling Of Exposed And Covered Surface Crack Detection Using Open-Ended Waveguides," Ph.D. Dissertation, Electrical Engineering Department, Colorado State University, Spring 1996.